Theoretical and Experimental Investigation of Asymmetric Coplanar Waveguides

VICTOR FOUAD HANNA AND DOMINIQUE THEBAULT

Abstract —**Conformal mapping techniques are used to obtain analytic closed-form expressions for the characteristic impedance and the relative effective dielectric constant of asymmetric coplanar waveguide with infinite or finite dielectric thickness. The line asymmetry leads to a decrease of its characteristic impedance and to an increase of its relative effective dielectric constant. Six asymmetric coplanar waveguides are realized and their characteristic impedances are measured using time-domain reflectometry techniques. The experimental results show very good agreement with the theoretical ones.**

I. Introduction

RECENTLY, symmetric coplanar waveguides (CPW) [1] have been the subject of growing interest as they have presented a solution to technical and technological problems encountered in the design of microstrip and slot transmission lines due to their easy adaptation to external shunt element connections as well as monolithic circuits. Here, we study the asymmetric CPW shown in Fig. 1 ($d_1 \# d_2$). The study of such a line is important as it allows one to evaluate the actual characteristics of a CPW normally designed to be symmetric, but the fabrication of which is imperfect.

During the whole analysis, we assume the ground planes to be infinitely wide and the strips to have negligible thickness. Also, we assume that the air–dielectric interfaces can be dealt with as though perfect magnetic walls (that is, enforcing Neumann boundary conditions) were present in them. This assumption can be easily verified for small gaps; however, it is noticed that it leads to adequately accurate results even for fairly large ones.

The analysis is performed using a zero-order quasi-static approximation [1] in order to obtain to quasi-TEM-line parameters (characteristic impedance and relative effective dielectric constant). This means that the line dispersion characteristics are not taken into consideration in this analysis. However, as is shown in the dispersion analysis of symmetric CPW presented by Knorr and Kuchler [2], the CPW parameters are only slightly sensitive to variations of the frequency for CPW's with dimensions that do not exceed the substrate thickness h for nearly the whole microwave region (e.g., for $\epsilon_r = 11$, $f(\text{GHz})\cdot h(\text{mm}) \leqslant 20$).

Manuscript received April 27, 1984; revised July 25, 1984.
The authors are with the Centre National d'Etudes des Télécommunications (CNET), Centre Paris B, Division ETR, 92131 Issy Les Moulineaux, France.

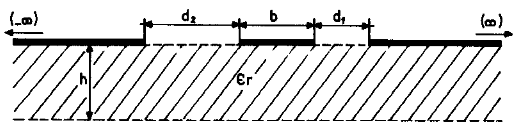

Fig. 1. Asymmetric coplanar waveguide.

II. Infinite Dielectric Thickness Asymmetric CPW

We [3] have treated this case in detail using conformal mapping techniques. We give here only the results. The characteristic line impedance Z_0 and the total line capacitance per unit length C can be written in the form

$$Z_0 = \frac{30\pi}{\sqrt{(1+\epsilon_r)/2}} \cdot \frac{K'(k_1)}{K(k_1)} \tag{1}$$

$$C = 2\epsilon_0(1+\epsilon_r)\left[K(k_1)/K'(k_1)\right] \tag{2}$$

where $K(k)$ is the complete elliptical integral of the first kind and $K'(k) = K(k')$, $k' = (1-k^2)^{1/2}$, and

$$k_1 = \frac{\frac{1}{2}b\left[1+\alpha\left(\frac{1}{2}b + d_1\right)\right]}{\frac{1}{2}b + d_1 + \alpha\left(\frac{1}{2}b\right)^2} \tag{3}$$

$$\alpha = \frac{d_1 d_2 + \frac{1}{2}b(d_1+d_2) \pm \left[d_1 d_2 (b+d_1)(b+d_2)\right]^{1/2}}{\left(\frac{1}{2}b\right)^2 (d_1 - d_2)}. \tag{4}$$

III. Finite Dielectric Thickness Asymmetric CPW

In this case, we assume that the total line capacitance per unit length is equal to the sum of the line capacitance per unit length in free space when the dielectric is replaced by air C_1 and the line capacitance per unit length C_2 obtained when assuming that the electric field is concentrated in a dielectric of thickness h and relative permittivity $(\epsilon_r - 1)$. This assumption has shown an excellent accuracy in the case of symmetric CPW [4]. So C_1 can be obtained from (2) by putting $\epsilon_r = 1$, i.e.,

$$C_1 = 4\epsilon_0 \frac{K(k_1)}{K'(k_1)}. \tag{5}$$

The line capacitance C_2 can be computed through a sequence of three intermediate conformal mappings (see

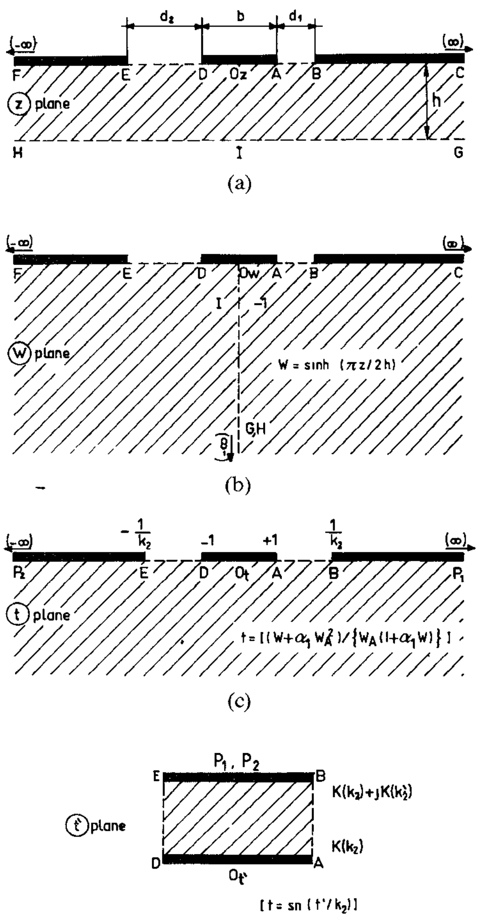

Fig. 2. Conformal transformations for calculating C_2 for an asymmetric CPW with finite h.

Fig. 2(b)–(d))

$$W = \sinh\left(\frac{\pi z}{2h}\right) \tag{6}$$

$$t = \frac{W + \alpha_1 W_A^2}{W_A(1+\alpha_1 W)} \tag{7}$$

$$t = \mathrm{sn}(t'/k_2) \tag{8}$$

where $\mathrm{sn}(t/k)$ is the sine elliptic function.

We have to notice that the conformal transformation of (7) is used to transform the asymmetrical boundary value problem of the W plane into a symmetrical one in the t plane. Here, α_1 and k_2 can be obtained from the symmetry condition

$$t_B = -t_E = \frac{1}{k_2}. \tag{9}$$

Then, C_2 can be given by the relation

$$C_2 = 2\epsilon_0(\epsilon_r - 1)\frac{K(k_2)}{K'(k_2)}. \tag{10}$$

Finally, the relative effective dielectric constant ϵ_{eff} and the characteristic impedance Z_0 of the asymmetric CPW

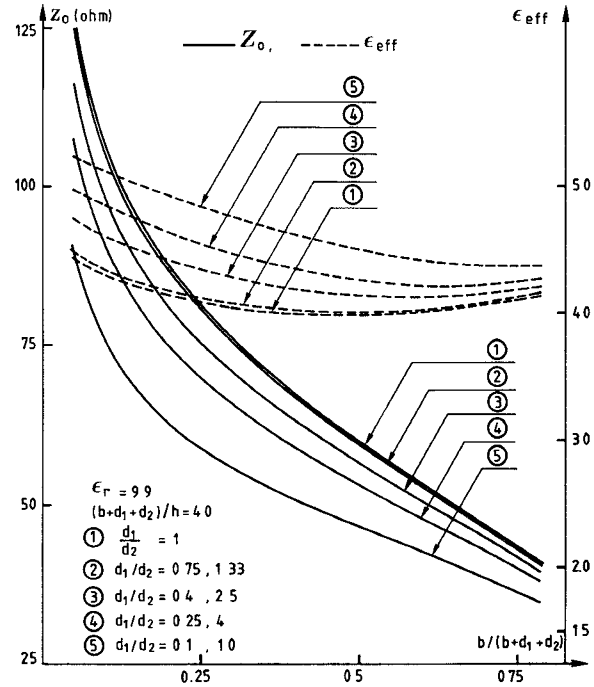

Fig. 3. Variation of Z_0 and ϵ_{eff} as a function of $b/(b+d_1+d_2)$ taking d_1/d_2 as a parameter for a CPW with $b+d_1+d_2 = 4h$.

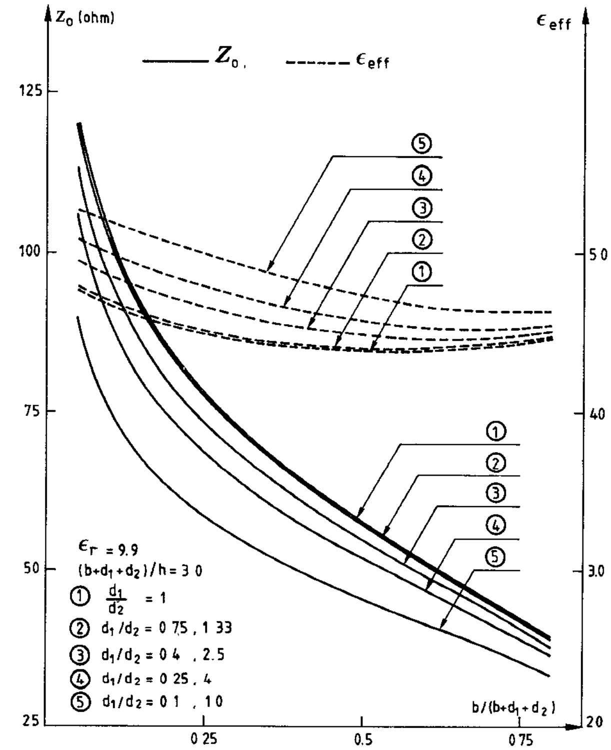

Fig. 4. Variation of Z_0 and ϵ_{eff} as a function of $b/(b+d_1+d_2)$ taking d_1/d_2 as a parameter for a CPW with $b+d_1+d_2 = 3h$.

can be written as

$$\epsilon_{\mathrm{eff}} = \frac{C_1 + C_2}{C_1} = 1 + \frac{1}{2}(\epsilon_r - 1)\frac{K(k_2)}{K'(k_2)}\cdot\frac{K'(k_1)}{K(k_1)} \tag{11}$$

$$Z_0 = \frac{30\pi}{(\epsilon_{\mathrm{eff}})^{1/2}}\frac{K'(k_1)}{K(k_1)} \tag{12}$$

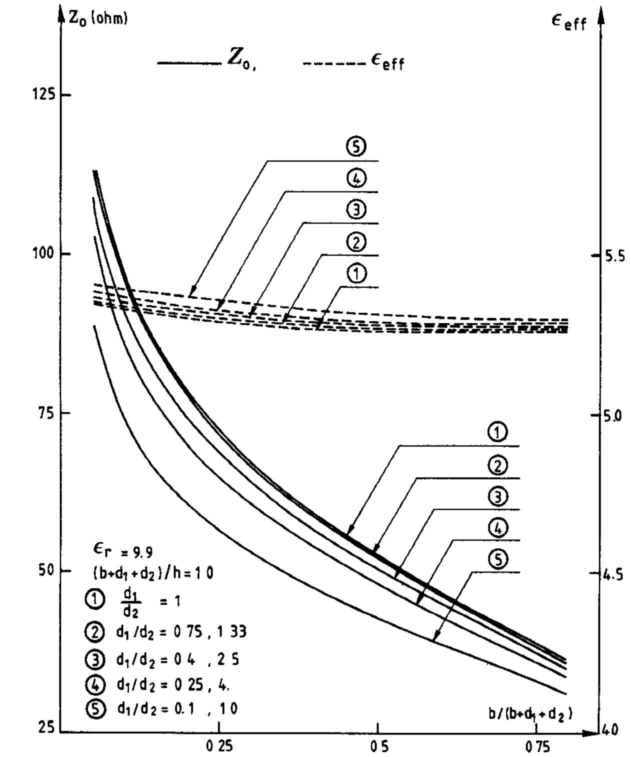

Fig. 5. Variation of Z_0 and ϵ_{eff} as a function of $b/(b+d_1+d_2)$ taking d_1/d_2 as a parameter for a CPW with $b+d_1+d_2=h$.

where

$$k_2 = \frac{W_A(1+\alpha_1 W_B)}{W_B+\alpha_1 W_A^2} \tag{13}$$

$$\left.\begin{aligned} W_A &= \sinh\left(\frac{\pi b}{4h}\right), W_B = \sinh\left[\frac{\pi}{2h}\left(\frac{b}{2}+d_1\right)\right] \\ W_E &= -\sinh\left[\frac{\pi}{2h}\left(\frac{b}{2}+d_2\right)\right] \end{aligned}\right\} \tag{14}$$

$$\alpha_1 = (W_B+W_E)^{-1} \cdot \left[-1-\frac{W_B W_E}{W_A^2} \pm \left\{\left(\frac{W_B^2}{W_A^2}-1\right)\left(\frac{W_E^2}{W_A^2}-1\right)\right\}^{1/2}\right]. \tag{15}$$

Hence, the line parameters can be calculated from (11) and (12) using the simple formulas of Hilberg [5] for the ratio $K(k)/K'(k)$.

Examples of design curves are given in Figs. 3, 4, and 5.

From our calculations, it can be concluded that, for a given shape ratio $b/(b+d_1+d_2)$, the line asymmetry leads to a decrease of its characteristic impedance and to an increase of its relative effective dielectric constant. However, these variations are not significant for asymmetry factors up to 25 percent (the asymmetry factors can be defined as $s=1-(d_1/d_2)$ if $d_2 \geqslant d_1$ or $s=1-(d_2/d_1)$ if $d_2 < d_1$). From Figs. 4 and 5, it can be seen that the line characteristic impedance and its relative effective dielectric constant differ by less than 2 and 4 percent, respectively, when the thickness of the substrate is reduced from infinity to $(b+d_1+d_2)$ and by less than 10 and 20 percent, respectively, when h is reduced to one third of $(b+d_1+d_2)$. So it is important to not neglect the effect of the finite dielectric thickness when the condition $h \geqslant (b+d_1+d_2)$ is not satisfied.

TABLE I
PARAMETERS OF THE SIX REALIZED ASYMMETRIC CPW'S

Line Number	d_1 (μm)	d_2 (μm)	b (μm)	$Z_0\|_{h=\infty}$ (ohm)	$Z_0\|$ h = finite (ohm)	$Z_0\|$ measured (ohm)
1	123	1 060	747	48.63	51.78	51.5
2	257	991	737	55.13	59.88	57.5
3	356	843	735	57.33	62.96	61.1
4	196	1 756	1 250	48.18	53.53	52
5	406	1 548	1 248	54.02	62.38	62.4
6	575	1 386	1 244	56.69	67.17	66.3

IV. EXPERIMENTAL RESULTS

Six asymmetric CPW's have been realized. Each was fabricated on an alumina substrate ($\epsilon_r = 9.9$ and $h = 0.635$ mm) metallized with gold of thickness 4 μm. The characteristic impedances of these lines are measured using time-domain reflectometry techniques, and they are calculated firstly using (1) ($h=\infty$) and secondly using (12) ($h=$ finite value). The lines dimensions, as well as the theoretical and experimental results, are given in Table I. The measurements are performed on an HP1817A time-domain reflectometer with a 35-ps rise-time pulse, which means that these measurements are valid for a frequency range from dc to 9.85 GHz.

The experimental results of Table I are in very good agreement with the theoretical ones and show the importance of not neglecting the substrate thickness when the condition $h \geqslant (b+d_1+d_2)$ is not fulfilled.

V. CONCLUSION

Analytic closed-form expressions for the characteristic impedance and the relative effective dielectric constant for asymmetric CPW with finite or infinite substrate thickness are obtained using conformal mapping techniques. Their accuracy has been verified experimentally.

Our results show that the CPW characteristic impedance and its relative effective dielectric constant vary slowly with onset of asymmetry and that it is important to not neglect the effect of the finite dielectric thickness for CPW's whose substrate thickness is smaller than the distance between their ground planes.

REFERENCES

[1] C. P. Wen, "Coplanar waveguide: A surface strip transmission line," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 1087–1090, Dec. 1969.

[2] J. B. Knorr and K.-D. Kuchler, "Analysis of coupled slots and coplanar strips on dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 541–548, July 1975.

[3] V. Fouad Hanna and D. Thebault, "Analysis of asymmetrical coplanar waveguides," *Int. J. Electron.*, vol. 50, pp. 221–224, Mar. 1981.

[4] V. Fouad Hanna and C. Veyres, "Extension of the application of conformal mapping techniques to coplanar lines with finite dimensions," *Int. J. Electron.*, vol. 48, pp. 47–56, July 80.

[5] W. Hilberg, "From approximations to exact relations for characteristic impedances," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 259–265, May 1969.